Chapter 2
Filamentation Physics

2.1 Some Experimental Observations

The basic physics of filamentation is universal and occurs in all transparent media (gases, liquids and solids). After propagating through an optical medium, a femtosecond (fs) Ti:sapphire laser pulse (at around 800 nm) turns into a white light laser pulse whose transverse pattern shows a central white spot surrounded by colored rings. The only difference is that the length of the filament is different in different media while the free electron generation mechanisms inside the filament core are different between gases and condensed matter materials. Figure 2.1a–d show the evolution of the transverse patterns of a 5 mJ/45 fs/800 nm Ti:sapphire laser pulse after propagating and filamenting in air without external focusing. The transform limited pulse from the vacuum compressor propagates into a 10 m vacuum pipeline which is connected directly to the vacuum compressor (see Fig. 2.2). After passing through the 1 cm-thick CaF$_2$ exit window, the pulse enters the corridor next to the author’s laboratory. It then passes through an inverted telescope so as to reduce the beam diameter from about 3.5 cm to about 6 mm. The diameter is measured at the $1/e^2$ level of the fluence distribution of the beam pattern on a white paper taken by a CCD camera. The colors in the single-shot-pictures in Fig. 2.1 are real. They are patterns of the pulse intercepted on a piece of white paper and are taken by a digital camera. The distance $z$ in the picture indicates the distance to the white paper screen from the exit of the inverted telescope (Fig. 2.2). This is a manifestation of single filamentation of the pulse that self-transforms into a white light laser pulse. The colored rings are conical emissions.

When the energy of the pulse becomes higher, multiple filaments occur. Figure 2.3a–f shows a series of similar patterns of the Ti:sapphire laser pulse at 50 mJ/45 fs propagating in air without the inverted telescope. The initial beam diameter at $1/e^2$ level of the fluence distribution is around 3.5 cm. The distance $z$ indicated in the figure is the distance measured from the beam steering optics (high reflectivity mirrors, see Fig. 2.2) which is set just outside the exit window of the pipeline. Because of the shot-to-shot fluctuation, the single shot patterns do not reproduce themselves, but the general trend of change is obvious. Each hot spot tends to self-focus into a white light pattern but there is also a filament competition process.
Fig. 2.1  Single filamentation. These are pictures taken by a normal digital camera.

among the hot spots to become white. This will be discussed later in the book under filament competition. The colored star like lines in Fig. 2.3e and f indicate the interference between fields of conical emissions originated from different self-focal spots (filaments). We shall later discuss how such a star like object can be created.
2.1 Some Experimental Observations

Colored pictures

Fig. 2.3 Multiple filamentation. The horizontal scale of each picture frame is 20 cm. These are colored pictures taken by a normal digital camera.

Such colored patterns can also be easily observed from the propagation in a condensed medium using a much lower laser power. Figure 2.4 shows an extreme case of propagating an unfocused Ti-sapphire laser pulse at 8 mJ/40 fs (diameter ~ 6 mm at FWHM of the fluence distribution) through a piece of 4 mm thick BK7 window. The peak power of the light is about 0.2 TW. This is about 5 orders of magnitude higher than the critical power for self-focusing in glass (2–3 MW). It thus generates

Fig. 2.4 Colored pattern of a 0.2 TW/40 fs Ti-sapphire laser pulse after propagating through a piece of 4 mm thick BK7 glass.
a large number of closely spaced filaments each becoming a white light laser source. Each source evolves into a pattern similar to that shown in Fig. 2.1c, d. The ensemble of these tightly spaced white light sources thus shows a more diffused set of colored rings (conical emission) with no resolution.

Now, when we look at the filament from the side in air, we can barely see a faint white line. Figure 2.5 shows a picture taken with an ICCD (intensified CCD) camera from the side of the filament which is the result of the propagation of a 45 fs Ti-sapphire laser pulse in air in a clean laboratory environment (class 100,000). The fine line of light comes mainly from the fluorescence of nitrogen molecules (Talebpour et al., 2000, 2001; Becker et al., 2001a). The diameter of the line is less than 100 μm. This is a manifestation of a single filament; but the line is not uniform. It shows a series of brighter sections separated by darker zones. This is a manifestation that the laser pulse undergoes multiple refocusing, generating a few filaments along the same propagation axis.

![Fig. 2.5 Side view of a filament in air. The diameter is 70 μm. It shows re-focusing](image)

If we use a sensitive burn paper to intercept the pulse at different positions of the propagation, we will observe a pattern similar to the one shown in Fig. 2.6. The central black spot is the so-called self-focal spot and the succession of such spots (self-foci) along the propagation axis gives rise to the perception of a filament. Because inside this hot spot, the intensity is high (about $5 \times 10^{13}$ W/cm²; see Kasparian et al., 2000a), nitrogen molecules are tunnel ionized. Tunnel ionization and multiphoton ionization are highly nonlinear processes that can be observed only at high laser intensities (see for example: Chin, 2004 and references therein). Thus, the low intensity pedestal outside the hot spot would not give rise to any measurable ionization signal. After the laser pulse has passed, the ionized molecules in the series of hot spots (self-foci) relax through collisions and fluoresce, giving rise to the picture shown in Fig. 2.5. It looks as if there was only a single filament (a series of hot spots) along the propagation axis; but in fact, there is a lot more radiation energy stored in the surrounding area which is normally not observed or omitted in
2.2 Experimental Definition of a Filament by Burn Paper

In the laboratory, it is easiest to use a burn paper to show the existence of a filament. This technique is at best ambiguous. This is because the damage threshold of the burn paper is low requiring normally the absorption of 3 to 4 photons whose intensity threshold is around $10^{10}$–$10^{11}$ W/cm² or less. However, ionizing air molecules takes about 8 photons and the intensity is clamped around $5 \times 10^{13}$ W/cm². Thus, soon after self-focusing starts and well before ionization takes place, the burn paper already shows a burn mark on the surface. After the end of filamentation, the peak power is lower than the critical power for self-focusing. There will thus be no collapse of the pulse and there will be no more ionization. The filament core, now being a single fundamental mode because of self-spatial filtering (see later), starts to diffract out. In the first part of this propagation, the intensity is still high enough to self-focus but is overcome by diffraction in such a way that the divergence of the pulse is very small. This would still create an impact on the burn paper. Thus, the definition of a filament depends very much on the sensitivity of detection of intensity inside the propagation zone, if we use intensity as the criterion. We shall come back to this subject later more quantitatively when we describe the full evolution of a filament.

**Fig. 2.6** Burn paper pattern. The *central dark spot* is the filament core (from Chin et al., 1999)

many experiments. We call this surrounding zone the background energy reservoir (Mlejnek et al., 1998, 1999; Kandidov et al., 2003a, b).
2.3 Single Filamentation Physics

We discuss single filamentation using air as the propagation medium. In order to describe the basic physics of filamentation (see for example, Boyd, 2003; Brodeur et al., 1997; Chin, 2006; Chin et al., 1999, 2005, 2008), we consider a short laser pulse, say, 50–100 fs in duration, from the most popular Ti-sapphire laser system (central wavelength 800 nm).

2.3.1 Slice-by-Slice Self-Focusing

For self-focusing to occur, the transverse spatial intensity distribution of the pulse across the wave front should not be uniform. We approximate the pulse as a plane-wave pulse and assume that the intensity distribution across the pulse’s transverse cross section is Gaussian. We shall follow the propagation of the central (most powerful) “slice” of the pulse. The thickness of this “slice” is at least $c\tau$, where $c$ is the speed of light in vacuum and $\tau$ is the period of oscillation of the electromagnetic wave. This is because we are talking about an intensity that is defined as the Poynting vector averaged over at least one cycle of oscillation. The propagation of this slice is similar to that of a wave front. If the intensity at the central zone of the slice is high enough so that the nonlinear Kerr effect cannot be neglected, the index of refraction of the central zone will be given by $n = n_0 + n_2I$, while the index at the edge of the slice will be $n = n_0$. Here, $n_0$ is the linear index of refraction in air and $n_2I$ is the Kerr nonlinear index of refraction; $n_2$ and $I$ being the coefficient of the Kerr nonlinear index of refraction and the local intensity, respectively. The speed of propagation of the slice is given by $c/n$. Hence, the central part of the slice propagates slower than the rest of the slice, giving rise to a concave wave front as shown in Fig. 2.7. This is the beginning of self-focusing. However, this self-focusing effect is not sufficient to guarantee filamentation because there is always a linear diffraction of the pulse that will cause the pulse to diverge as it propagates further. If the self-focusing effect is not strong enough to counteract the diffraction effect, the consequence is a slowly divergent pulse, slower than that due to pure linear diffraction. Consequently, the pulse’s diameter looks almost constant over some distance of propagation.

When the natural linear diffraction of the pulse is just balanced by self-focusing, the peak power equals the so-called critical power for self-focusing. Through a solution of Maxwell’s equations for a nonparaxial CW Gaussian beam, the critical power for self-focusing is given by $P_c = \frac{3.77\lambda^2}{8\pi n_2n_0}$, where $\lambda$ is the central wavelength of the pulse (Marburger, 1975). This expression shows that the critical power for self-focusing depends only on $n_2$, $n_0$ and $\lambda$ and is independent of the intensity. Thus, when the peak power of the pulse is higher than the critical power for self-focusing, the slice shown in Fig. 2.7 will continue to curve forward as the wave front propagates further. If the peak power is only very slightly higher than $P_c$, the group
velocity dispersion (GVD) will lengthen the pulse after a short distance of propagation; this lowers the peak power to a value below $P_c$ and the pulse will again diverge slowly through diffraction. However, with femtosecond laser pulses, it is easy to obtain a high peak power that can readily overcome both linear diffraction and GVD. A few tens of percent higher than $P_c$ are enough (Brodeur and Chin, 1998, 1999). Once such self-focusing starts, it will not stop. Thus, the slice keeps curving into a smaller and smaller zone as it propagates while the intensity becomes higher and higher (Fig. 2.7). Soon, the high intensity in the self-focal zone will tunnel ionize (see Chapter 1 and Chin et al., 1985; Chin, 2004) air molecules, resulting in the generation of a weak plasma. The change in the index of refraction of the slice propagating in a plasma is\(^1\) \( (\Delta n)_p \cong -\frac{4\pi e^2N_e(t)}{2m_e\omega_0^2} \), where $N_e$ is the electron density.

\(^1\)The contribution to the index of refraction by a plasma can be obtained in the following way. In a plasma, from any optics text book, by assuming that only the electrons contribute, the index of refraction is given by

\[
n^2 = 1 - \frac{\omega_p^2}{\omega^2}
\]

where the subscript $p$ denotes plasma; the plasma frequency $\omega_p = [4\pi e^2N_e(t)/m]^{1/2}$ (where $e$ and $m$ are the electron charge and mass in cgs units, respectively and $N_e(t)$ is the MPI/TI generated time dependent density in $\text{cm}^{-3}$, i.e., $N_e(t)$ depends on the intensity of the laser). In air, $\omega_p << \omega$. This is always the case in the self-focus where the intensity is clamped down to between $10^{13}$ and
$e$ and $m_e$ are the electronic charge and mass, respectively, and $\omega_0$ is the central frequency of the pulse. The index of refraction of the central part of the slice is, thus $n = n_0 + n_2 I - \frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$. This will increase the speed of propagation of the central part of the slice; i.e., the curvature of the slice starts to flatten out, but it is still focusing so long as $n_2 I > \frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2}$. Thus, the intensity is still increasing. The electron density increases very rapidly with the intensity because tunnel ionization is a highly nonlinear process. We approximate such an increase as being governed by an effective power law according to an experimental observation (Talebpour et al., 1999a); i.e., $N_e(t) \propto I^m$, where $m$ is the effective nonlinear order of ionization. In air, $m$ is about 8 (Talebpour et al., 1999a). The effective index of refraction of the central part of the slice is thus $n = n_0 + n_2 I - \frac{4\pi e^2 k I^m}{2m_e\omega_0^2}$, where $k$ is a proportionality constant. Qualitatively, this means that the free electron term will quickly catch up with the Kerr term until they are equal; i.e., until $n_2 I = \frac{4\pi e^2 k I^m}{2m_e\omega_0^2}$. At this point, Kerr self-focusing balances free electron defocusing, and the central part having now an index of refraction $n_0$ propagates at the same speed as the rest of the slice. There is no more focusing (i.e., a local plane wave front, see Fig. 2.7) and the intensity is highest at this balancing point. This is the condition of intensity clamping (Kasparian et al., 2000a; Becker et al., 2001; Liu et al., 2002) because further propagation would lead to an index at the central part smaller than $n_0$. The slice will start to diverge. That is to say, during self-focusing of a powerful femtosecond laser pulse in an optical medium, there is a maximum intensity that self-focusing can reach. In air, it is around $5 \times 10^{13}$ W/cm$^2$ (Becker et al., 2001). The energy in the defocusing slice will be reduced a little due to the loss in ionization. After passing through the self-focus, the central slice is returned (defocused) back to the remaining part of the whole pulse or to the background reservoir (Mlejnek et al., 1998, 1999; Kandidov et al., 2003a). This background reservoir is an important concept in considering the physics of filamentation. An experimental and numerical study of the background reservoir is given in (Liu et al., 2005b) and will be discussed later in the book.

$10^{14}$ W/cm$^3$ (Théberge et al., 2006) at which single ionization dominates. The electron density in a filament in air generated by a 50 fs/800 nm laser pulse has been measured to be of the order of $10^{16}$/cm$^3$. This gives $\nu_p = \omega_p/2\pi = 3 \times 10^9$ Hz which is much smaller than the optical frequency ($\sim 10^{14}$ Hz). Hence, Eq. (f2.1) becomes

$$n = \left[1 - \frac{\omega_p^2}{\omega^2}\right]^{1/2} \cong 1 - \frac{\omega_p^2}{2\omega^2} \quad \text{(f2.2)}$$

When $N_e(t) = 0$, i.e., in vacuum, $n = 1$. That is to say, $n = 1 + \Delta n_{\text{plasma}}$ where $\Delta n_{\text{plasma}}$ is the contribution of the plasma to the index of refraction.

$$\Delta n_p = -\frac{\omega_p^2}{2\omega^2} = -\frac{4\pi e^2 N_e(t)}{2m_e\omega_0^2} \quad \text{(f2.3)}$$

which is negative.
The critical power for self-focusing, which is inversely proportional to the coefficient of the nonlinear index of refraction, \( n_2 \), is not constant in air. It depends on the response of the medium to the pulse duration. We note that the response of a medium to an electromagnetic wave is essentially the induced polarization (dipole moment per unit volume). When the pulse duration is shorter than 100 fs, only an “instantaneous” electronic response (induced polarization due to a pure electronic oscillation that can follow the field) is fast enough to contribute to the total (linear and nonlinear) polarization, which in turn contributes to the total index of refraction and to \( n_2 \). When the pulse is longer so that the interaction time is longer, both the electronic and the nuclear responses involving the Raman transition (excitation of a molecular vibration) contribute to a larger value of \( n_2 \). This lowers the critical power. In air, our recent experimental measurement (Liu and Chin, 2005) shows that \( P_c \) changes from about 10 GW for pulse durations shorter than 100 fs to about 3 GW for pulses longer than 100 fs.

The plasma density in the self-focus depends on the external condition. For free propagating beam in one atmospheric air, it is measured to be around \( 10^{14} \text{ cm}^{-3} \) (Théberge et al., 2006a). This value increases when using an external focusing lens. The shorter the focal length is, the higher the density will be. The highest measured value is of the order of \( 10^{18} \text{ cm}^{-3} \) using a 10 cm focal length lens in air (Théberge et al., 2006a).

The central slice will self-focus at a self-focusing distance \( z_f \) from the beginning of the propagation in the medium given by

\[
\frac{0.367k a_0^2}{\left\{ \left( \frac{P}{P_c} \right)^{1/2} - 0.852 \right\}^2 - 0.0219} \right\}^{1/2} (2.1)
\]

where \( k \) is the wave number, \( a_0 \), the radius of the beam profile at \( 1/e \) level of intensity, and \( P \), the peak power of the slice. The slice in front of the central slice will then self-focus at a later position in the propagation direction according to Eq. (2.1) because its peak power is lower than that of the central slice. It will undergo the same processes, namely, self-focusing, intensity clamping, and de-focusing, and will return the (slightly lowered) energy back to the background reservoir, and so on for successive front slices whose peak powers are higher than the critical power (Figs. 2.8 and 2.9). Thus, the front part of the pulse will become thinner and thinner as the

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2Equation (2.1) is the solution of the nonlinear Schröedinger equation coming from the Maxwell’s equations without GVD. The initial conditions are: (1) the laser beam is a continuous one (CW); (2) it is a paraxial cylindrical beam with a spatial Gaussian distribution of intensity at the input of the medium; (3) slowly varying envelope approximation is used. During the propagation, the beam is deformed and numerical technique is used to obtain Eq. (2.1). Interestingly, this equation was found to be applicable even down to pulse duration of about 10 cycles of oscillation of the field. For example, in the case of a Ti-sapphire laser at the wavelength of around 800 nm with a pulse duration of the order of 100 fs, this equation was found to describe well the beginning of filamentation (Brodeur et al., 1997).
Fig. 2.8 Slice-by-slice self-focusing in an ideal case if there is no ionization at the self-focus. The top part shows the pulse represented by a distribution of slices. The central most powerful slice self-focuses at a shorter distance in the direction of propagation z. The weaker slices at the side self-focus at longer distances. The lower part shows an ellipsoid that represents the pulse. The vertical dimension is power and the horizontal dimension is spatial length of the pulse. The ellipsoid is squeezed down as it propagates into successive structures shown at the right side of the ellipsoid. However, in reality, ionization takes place at the self-focus. The front part of the pulse (ellipsoid) will still undergo the same squeezing, while the back part will squeeze into the ionization zone of the preceding slice. This will induce interaction with the plasma. The consequence is shown in Fig. 2.9

Fig. 2.9 A schematic diagram of the evolution of a femtosecond laser pulse propagating in an optical medium. The direction of propagation is towards the right hand side. The pulse is represented by the ellipse at the left. The width of the ellipse represents the pulse’s spatial width (c times the pulse length). This width is of the order of 30 μm for a 100 fs pulse; hence, a sheet of light. As such, the ellipse could be imagined as a “pancake” of light with a lot of photons inside. The central slice of the pulse self-focuses to a small area where the resulting high intensity ionizes the air molecules (star). The front part keeps on self-focusing, becoming thinner and thinner. The back part encounters the plasma left behind by the front part and undergoes self-phase modulation; the resultant intensity (field) distribution becomes non-uniform and is represented by the diverging “splashes” of elongated “sub-pancakes” of light. At the end of the propagation, the pulse degenerates into a colorful white light laser pulse (from Chin et al., 2005)
pulse propagates. The back slices symmetrical to the front slices will, in principle, also self-focus at a position slightly behind the self-foci of the front slices (Fig. 2.8). However, this will never happen because it will encounter the plasma left behind by the central and successive front slices. These back slices will thus self-focus into and interact with the plasma giving rise to a complex intensity distribution (see for example, Kandidov et al., 2003a; Aközpek et al., 2000). In general, the energy in the back part of the pulse will still be confined inside the highly deformed body of the pulse or the background reservoir (Fig. 2.9).

During the propagation, repeated processes of Kerr self-focusing in the neutral gas and self-defocusing in the self-generated weak plasma of the slices in the front part of the pulse result in a continuous series of hot spots along the propagation axis. This gives rise to the perception of a filament, and hence, filamentation (Brodeur et al., 1997; Kosareva et al., 1997a; Mlejnek et al., 1998; Chiron et al., 1999) (Fig. 2.9). Since the energy loss in the ionization process is small, the pulse can repeat the whole process again, resulting in what we call self refocusing (Mlejnek et al., 1998; Talebpour et al., 1999b). This is manifested by successive sections of brighter lines in Fig. 2.5.

Before ending this section, the author would like to comment upon the so-called Marburger formula; i.e., Eq. (2.1). Very often, one says that $z_f$ is independent of intensity because the formula seems to indicate that $z_f$ is not a function of the intensity of the pulse (slice). However, a closer look reveals an implicit dependence on the intensity through the radius $a_0$. This is the radius of the intensity distribution of a pulse with spatial Gaussian distribution. It is related to the intensity through $I_p = P_p / \pi a_0^2$ where $P_p$ and $I_p$ are the peak power and peak intensity, respectively. Thus, for a constant pulsed energy with a constant pulse duration (i.e., a constant peak power), changing the radius automatically changes the peak intensity or vice versa. This in turn changes the value of $z_f$, i.e., $z_f$ also depends on the intensity.

### 2.3.2 Intensity Clamping

Intensity clamping is a profound physical manifestation of self-focusing and filamentation. It sets an upper limit to the intensity at the self-focus not only in air but also in all optical media. Even if one tries to focus the pulse, so long as the focal length is not too short (Liu et al., 2003), self-focusing will always start before the geometrical focus. Thus, the intensity at the geometrical focus is either lower than, or as high as that inside the self-focal zone in air. The consequence of this intensity clamping is far reaching. In air, one can have self-focusing at a long distance but one cannot further increase the intensity inside the self-focus, not even by significantly increasing the energy of the pulse to many times the critical power. In practice, there will only be an increase in the number of self-foci (multiple filamentation), each of which will have similar peak intensity. The “dream” of reaching an enormous intensity (that might induce a nuclear reaction, for example) on remote targets in the atmosphere has to be forgotten in the current context. On the other hand, if
the beam profile is so smooth that only a single filament will persist while the peak power is increased significantly to many times the critical power for self-focusing, the diameter, and hence the volume of the filament will increase while the intensity inside this larger volume will still be clamped (Théberge et al., 2007a). Also, multiple re-focusing will take place. In practice, this is a tough condition to fulfill because any little fluctuation in intensity on the beam profile will lead to local self-focusing so long as the local power is higher than the critical power for self-focusing. This again results in multiple filaments. Furthermore, because the intensity is almost constant, any interaction making use of, or sampling the filament core, will result in a very stable outcome. One example is third harmonic generation (Aközbek et al., 2002).

The clamped intensity in air (or gases) is independent of pressure. Thus, when filamentation occurs at a high altitude in the atmosphere, the clamped intensity is always the same as that at sea level because when intensity clamping occurs, the nonlinear Kerr index change and the index due to plasma generation are equal:

\[ n_2 I = \frac{4\pi e^2 N_e(t)}{2m_e c^2 \omega_0^2}. \]

Both \( n_2 \) and \( N_e(t) \) are linearly proportional to the gas density since \( N_e(t) \) comes from tunnel ionization of the individual molecules. Hence, the gas density cancels out on the two sides of the equation, leaving behind an equation for the solution of the same clamped intensity \( I \) at any pressure. This claim is verified experimentally by the author and collaborators (Bernhardt et al., 2008) using He as the target gas because of its simplicity and unambiguity.

One might ask if GVD and linear diffraction could also have led to intensity clamping since they both would be able to stop self-focusing from developing. In linear diffraction, if the peak power is smaller than the critical power, there will never be a strong focal spot in the propagation. The beam will keep on growing in diameter. When the peak power equals the critical power, the beam will self-focus at infinity. (This is the definition of the critical power.) If the peak power is increased further, self-focusing overcomes linear diffraction and will self-focus towards a singular point. If nothing else happens, ionization will occur and the intensity is clamped. However, if GVD is large; i.e., if the bandwidth of the pulse is large, different frequencies in the pulse will propagate at different group velocities leading to an elongation of the pulse. Consequently, if the initial peak power is not high enough, it could become lower than the critical power. One could say that at the point where GVD and linear diffraction balance self-focusing, the intensity is clamped. But this balancing intensity depends on the bandwidth of the pulse (which can be changed at will, in principle) and will not lead to the occurrence of a series of hot spots (filament). Thus, we could not use this balancing point as the criterion of intensity clamping. When both linear diffraction and GVD are overcome by self-focusing, the beam will keep on focusing to a small spot until ionization occurs and balances self-focusing. It is this balancing point that is unique because it depends on the ionization potential (probability) of the molecules/atoms in the medium. The ionization potential of an atom or a molecule cannot be changed. It is fixed by nature rendering the clamped intensity unique.
2.3 Single Filamentation Physics

2.3.3 Is There Optical Breakdown During Filamentation?

Very often, when the idea of ionization at the self-focus is proposed to be the mechanism that balances the effect of self-focusing resulting in intensity clamping, the “instinctive” reflex of many experienced scientists would be that the ionization is naturally an optical breakdown. This is because it involves (self) focusing a strong laser pulse into a transparent optical medium. This would lead to the belief that the plasma generated by optical breakdown would stop further propagation of the laser pulse, as is the case for nanosecond and picosecond pulses (see Chapter 1). The peak laser power involved in such experiments can be a few MW (Chin and Isenor, 1967 and references therein). Such a phenomenon is shown in Fig. 2.10. The 10 ns second harmonic (532 nm) of a YAG-laser pulse is focused by a 50 cm focal length lens in air. The plasma grows towards the lens and is stronger and longer in front of the focus than behind. This is because once the optical breakdown threshold is reached during the rising part of the pulse, a plasma is created at the geometrical focus (see Fig. 2.11a). There is a lot of time for this plasma to absorb the rest of the laser radiation and grow and scatter the remaining part of the pulse. The self-focusing threshold is much higher than the breakdown threshold and hence will never be reached before the strong growth of the plasma. The consequence is that the length of the plasma cannot be very long. Thus, during self-focusing of an fs laser pulse, if the above mechanism did take place, we would never have observed a long filament. Since a long filament did occur, the ionization model should not be valid.

![Image](image.png)

**Fig. 2.10** Breakdown by focusing a 700 mJ/10 ns laser pulse at 532 nm in air by a 50 cm focal length lens. The plasma grows towards the laser while not much laser energy penetrated through the plasma, and the plasma is much weaker at and behind the geometrical focus. The total length of the plasma is of the order of 20 mm (picture taken by S. Mehdi Sharifi in the author’s laboratory)

As discussed in Chapter 1, the mean free time of a free electron is about 1 ps in atmospheric air, while usually filamentation experiments are done using pulses of a few tens of fs or longer. There is thus not enough time for the free electron to undergo collision with the surrounding molecules before the laser pulse is over.
Hence, even a first round of inverse Bremsstrahlung is not possible in atmospheric air. This is shown in Fig. 2.11b. The sharp rise of the pulse is such that well before the threshold of breakdown is reached, self-focusing takes place. After that, the plasma generation in the self-focal region is mainly direct tunnel ionization of the molecules.

Also, the slice-by-slice self-focusing mechanism does not allow the full temporal (spatial) part of the fs laser pulse to focus into the same focal volume. Each slice self-focuses at a difference zone in space. This slice is thus shorter than the full pulse width. The interaction time for ionization is thus shorter than the full pulse duration resulting in less ionization or the generation of less plasma. Moreover, since the pulse undergoes self-compression in time during filamentation, the interaction time would be of the order of one or two cycles of field oscillation. New interaction physics is expected.

In condensed matter, because the mean free time is about 1 fs, there would be sufficient time to induce breakdown. However, because of the slice-by-slice self-focusing mechanism, the interaction time inside a slice is of the order of one or two cycles of field oscillation (see Section 2.3.1). This would mean that there is still not much time for the plasma to grow through full cascade ionization. The plasma density inside the filament in a condensed matter is thus in the order of $10^{18} - 10^{19}$ cm$^{-3}$ which is 3–4 orders of magnitude lower than the density of condensed matter. This is a major reason why femtosecond laser pulses create much less damage than nanosecond and picosecond laser pulses in optical media.
2.3.4 Effect of External Focusing

In many experiments, an external lens/mirror is used to force self-focusing within the limited length of the medium. Self-focusing is then reinforced by the external lens such that the self focus appears before the position of the geometrical focus. The new self-focusing distance \( z'_f \) satisfies the lens transformation equation:

\[
\frac{1}{z_f} + \frac{1}{f} = \frac{1}{z'_f},
\]

where \( z_f \) is the self-focusing distance given by Eq. (2.1) and \( f \) is the geometrical focal length of the lens/mirror (Talanov, 1970). Figure 2.12 (Liu et al., 2003) shows the measured fluorescence from nitrogen molecules from inside the filament regions using three different focal lengths. For long geometrical focal length, filamentation occurs before the geometrical focus (Fig. 2.12a) while the intensity at the geometrical focus is low. In such cases, the filament length is roughly the distance between the self-focusing position at \( z'_f \) and the geometrical focus. When the geometrical focal length becomes shorter, the filament extends into the geometrical focus (Fig. 2.12b). Here, the effect of geometrical focusing becomes evident. At the geometrical focus, the stronger external focusing generates a plasma stronger than that generated by the filament. Self-focusing and geometrical

![Figure 2.12](image)

**Fig. 2.12** The “competition” between self-focusing and geometrical focusing in air. *Images* show the measured fluorescence from nitrogen molecules from inside the weak plasma regions. They are taken via a grating’s zeroth order image; the 42 fs laser pulse propagates towards the top; input energy, 5 mJ/pulse. (a) \( f = 100 \) cm, (b) \( f = 30 \) cm, (c) \( f = 5 \) cm; *inset*: low contrast pictures of \( f = 30 \) cm and \( f = 5 \) cm. When the geometrical focal length is long, filamentation occurs before the geometrical focus (a). When the geometrical focal length becomes shorter, the filament extends into the geometrical focus (b). Self-focusing and geometrical focusing become indistinguishable when the geometrical focal length is very short (c) (from Liu et al., 2003)
focusing become indistinguishable when the geometrical focal length is very short (Fig. 2.12c).

Intensity clamping still occurs with geometrical focusing. However, the intensity would increase slightly while the plasma density would increase drastically. Qualitatively, when geometrical focusing is added to self-focusing, it is effectively similar to having a stronger focusing; i.e., a concave wave front with a larger curvature as compared to that of a self-focusing plane wave front. In order to overcome this stronger focusing, the self-generated plasma has to become denser so that the negative lensing effect can balance the focusing effect. To see more quantitatively how geometrical focusing influences the plasma density at the focus where intensity clamping occurs, we go back to Sections 2.3.1 and 2.3.2 as well as Fig. 2.7. The change in the index of refraction in the lens along the propagation axis with respect to the edge of the lens is $\Delta n_{\text{lens}}$. This should be added to the nonlinear increase of the index of refraction $\Delta n_{\text{Kerr}} = n_2 I$ and together, they are balanced by the self-generated plasma’s index of refraction (Chen, 2008, private communication) which is

$$
(\Delta n)_p \approx -\frac{4\pi e^2 N_e(t)}{2m_e \omega_0^2}
$$

i.e.,

$$
\Delta n_{\text{lens}} + n_2 I \approx \frac{4\pi e^2 N_e(t)}{2m_e \omega_0^2}
$$

It is evident that to reach this balance, $N_e(t)$ must be larger than when there is no geometrical focusing (free propagation); i.e., the intensity must be higher when an external geometrical focusing is used. Experimentally, it was observed by Théberge et al. (2006a). However, the increase in intensity is not very much even though the plasma density changes by two to three orders of magnitude. This is because the plasma is generated through the highly nonlinear multiphoton/tunnel ionization process. A little change in intensity would lead to a huge change in the plasma density.

### 2.3.5 Background Energy Reservoir

One important consequence of slice-by-slice self-focusing is the concept of the so-called background energy reservoir or energy reservoir or simply reservoir. A self-focusing slice will generate a plasma which defocuses the slice. The energy of this slice is thus not always confined along the propagation axis. It goes back to the surrounding region but is always a part of the pulse. After a round of slice-by-slice self-focusing of the whole pulse, a plasma column along the axis is left behind (filament).

Because the so-called filament or the series of self-foci are fed from outside the filament core, one has to be careful in using different words to describe the
same phenomenon. For example, self-channeling is used very often to describe filamentation. But the word “self-channeling” might implicitly be erroneously interpreted by the non-initiated to mean that it is the whole pulse that self-focuses and continues channeling its way through the medium as if it were a “light-bullet”. The latter name is also used from time to time for reasons of sensation. But in fact they all mean the same physics of slice-by-slice self-focusing.

The existence of a wide background reservoir has been noticed already in the first demonstrations of filamentation in air (Braun et al., 1995; Nibbering et al., 1996; Brodeur et al., 1997). It has been interpreted by results of computer simulations as an energy reservoir of the filament core (Mlejnek et al., 1998). Several experiments (Brodeur et al., 1997; Dubietis et al., 2004b; Liu et al., 2005a, b) showing that filaments are terminated after blocking the outer part of the pulse except its core, provided support for this interpretation. Recently, it has been further shown in an experiment (Courvoisier et al., 2003) and subsequent numerical simulations (Dubietis et al., 2004b; Kolesik and Moloney, 2004; Skupin et al., 2004) that filaments are robust after a collision with a water droplet. This observation has been attributed as being due to an energy transfer from the background to the filament core. Also, results of a variational analysis (Arevalo and Becker, 2005) indicate that already the process of self-focusing is strongly influenced by the wide background of the pulse. The following gives a detailed experiment that shows the existence of the background energy reservoir. The size of the reservoir and the portion of the pulse energy located in the background are also estimated (Liu et al., 2005b).

A Ti-sapphire laser was used. It gave out 800 nm/50 fs pulses with a spectral bandwidth of 23 nm at FWHM, a repetition rate of 10 Hz and energy of 2.5 mJ/pulse. The peak power is many times larger than the critical power of self-focusing in air (~6–10 GW) (Liu and Chin, 2005). The initial beam diameter has been reduced to 2 mm at FWHM by a telescope consisting of a pair of a plano-convex lens ($f = 40$ cm) and a bi-concave lens ($f = -20$ cm) (Fig. 2.13). Using this set-up, a long single filament was obtained in air.

![Fig. 2.13 Experimental set up to demonstrate the existence of background energy reservoir](image)

Pinholes of different diameters (220 μm to 2 mm) were introduced coaxially to the propagation axis of the beam. The evolution of the filament formation was observed by using an intensified CCD camera (ICCD, Princeton Instruments, PI-MAX 512) to image the fluorescence emitted from the nitrogen molecule ($\text{N}_2$) and nitrogen ion ($\text{N}_2^+$) in the filament core (Talebpour et al., 2001). The ICCD camera
was installed perpendicularly to the pulse propagation axis and the fluorescence signal was collected and imaged onto the ICCD detector by using a single plano-convex lens made of fused silica with a focal length of 63 mm and a diameter of 38.1 mm. A band-pass filter (1 mm-thick UG11, Corion) together with a 0° incident 800 nm dielectric mirror were placed in front of the camera to integrate the light emission over the strongest N$_2$ and N$_2^+$ bands around 350 nm while rejecting the scattered light from the pump laser (Hosseini et al., 2003). There is a good overlap between the band–pass filter transmission curve and the strongest N$_2$ and N$_2^+$ bands around 350 nm. With this configuration, about 0.5 m along the laser propagation axis was covered by the field of view of the ICCD detection system. Furthermore, a slim black screen (~ 3 cm wide) was put beside the pinhole to avoid strong scattering of the laser light from the metal surface of the pinhole to the detector.

Images acquired by the ICCD camera are shown in Fig. 2.14a–e. Each image results from an accumulation of 1,000 laser shots. The laser propagates from the left to the right and the propagation distance is scaled with respect to the concave lens of the telescope. Figure 2.14a represents the free propagation of the pulse (without pinhole). A bright line over almost half a meter is seen along the propagation axis. The onset of strong nitrogen fluorescence, and hence, the beginning of the filament is located near the left side of the image. The signal decays toward the right side of the figure after it has reached its maximum at a distance of about 1.7 m. The other panels on the left hand side of Fig. 2.14 show the images for pinholes of different diameters inserted at 1.73 m, starting with a diameter of 220 μm (Fig. 2.14b) and up to 2 mm (Fig. 2.14e). The black gap between 1.7 and 1.8 m in these four panels appears due to the slim black screen mentioned above. It is seen that for the smallest diameter
(Fig. 2.14b, 220 μm), the plasma column is terminated by the pinhole (the white spot results from diffraction by the pinhole). In this case, no significant damage of the pinhole was found after the experiment which indicates the high stability of the laser system. That ensures the successes of our experiments. As the diameter of the pinhole is doubled (Fig. 2.14c, 440 μm), the filament partially survives up to about 1.9 m. When the pinhole diameter is increased to 1 mm (Fig. 2.14d) or 2 mm (Fig. 2.14e), the filament formation seems to be unchanged compared to the case of free propagation within the field of view of the camera.

Numerical simulations of the amplitude envelope $A$ based on the nonlinear wave equation written in the retarded coordinate system (using the slowly varying envelope approximation) was performed. (The theory will be discussed later in the book. Readers not yet familiar with the theory can skip this part.)

\[
2ik_0 \frac{\partial A}{\partial z} = \Delta \mathbf{A} - k_2k_0 \frac{\partial^2 A}{\partial \tau^2} + 2 \frac{k_0^2}{n_0} (\Delta n_{Kerr} + \Delta n_{plasma})A - ik_0A \tag{2.2}
\]

In Eq. (2.2), diffraction, dispersion, and Kerr effect as well as plasma generation and energy losses due to multiphoton/tunnel ionization are considered. Values for $k_0, k_2, \Delta n_{Kerr}, \Delta n_{plasma}$ and $\alpha$ are adapted from Kandidov et al. (2003a) and parameters of the laser system (2.5 mJ, 40 fs, 2 mm diameter at FWHM) have been used as initial conditions in the simulations. The effects of the pinholes are simulated by applying an energy transmission function at 1.7 m. The transmission function is set to be unity from the axis to the corresponding pinhole radius and smoothed toward the outer edge by a Gaussian function with 10 μm ($1/e^2$) width.

Electron density distributions obtained from the simulations are shown in Fig. 2.14f–j. The reason why the fluorescence is compared to the electron density is the following. The nitrogen fluorescence is the result of multiphoton/tunnel ionization of nitrogen molecules. Thus, the electron density and the fluorescence are related to each other directly. Each panel in Fig. 2.14f–j corresponds to the experimental result in the same row. It is seen that the experimental results are well reproduced by the simulations. Also, for the percentage of initial energy transmitted through the pinhole (Fig. 2.15a), there is a good agreement between the experimental results (open triangles) and those from simulations (solid squares) for all pinhole diameters. The results for the on-axis electron density distribution from numerical simulation, shown in Fig. 2.15b, reveal differences in the long-scale propagation outside the view of the ICCD camera between the cases of the 1 mm pinhole and the 2 mm pinhole. The refocusing peak, which appears roughly between 2.2 and 3 m for free propagation (solid line), is suppressed for the smaller pinhole (1 mm, dashed line) but not for the pinhole of 2 mm in diameter (dotted line). Thus, a background whose transverse dimension is about 5–10 times as large as the filament core and containing up to 50% of the pulse energy (see Fig. 2.15a), has to propagate together with the filament core in order to maintain the full length of the filament including one refocusing.

The results suggest that a diffraction of energy at the edges of the background lead to a leakage of energy which results in a termination of the filament. This
interpretation is supported by the energy distributions obtained from the numerical simulation displayed in Fig. 2.16 for the cases of (a) free propagation, (b) with a pinhole of 1 mm in diameter and (c) with a pinhole of 2 mm in diameter. The horizontal and vertical axes correspond to the propagation distance and the radius respectively. Grey levels represent the energy percentage within a given radius $r_0$ about the propagation axis; the lighter the color, the larger the portion of energy enclosed. Four contour lines indicating the 10, 20, 50, and 80% levels are plotted to guide the eye. The bending of the line towards the propagation indicates a flow of energy towards the filament core and vice versa. The oscillations of the inner contour lines indicate the energy exchange between the filament core and the outer background (see Fig. 16a). Moreover, a large part of the pulse energy propagates together with the filament core before the energy starts to diffract out at the end of the filament. The pinholes initiate the diffraction at the edges of the background at an earlier stage of the filament formation compared to the case of free propagation (see Fig. 2.16b, c). For the smaller pinhole, this results in the termination of the filament before refocusing takes place.
The above results have important implications for long-range filament formation. It is obvious that for the maintenance of the high-intensity filament core which contains about 10% of the pulse energy only, the propagation of a wide background, in which 50% or more of the pulse energy is located together with the core, is needed. For the filament formation it is obviously more critical to avoid a diffraction of energy at the edges of the background than a collision with a (small) droplet near the center. While the former scenario appears to lead to a continuous leakage of energy towards the outside resulting in the termination of the filament sooner or later, the latter may just alter the energy flow within the core and the background. We may finally note that our results clearly rule out any self-guiding model for the filament core but do favor the moving focus (Brodeur et al., 1997) and spatial replenishment (Mlejnek et al., 1998) models. They may however raise the question if the whole filament, i.e., core and background, could be described by spatial soliton solutions or a superposition of such solutions (Dubietis et al., 2004b; Courvoisier et al., 2003).

The above shows that the formation of robust filaments is due to the wide low-intensity background around the tiny high-intensity core. We have shown that this background is more than 5 times larger than the filament core and contains up to 50% of the pulse energy. The robustness of the filament formation does depend crucially on diffraction of energy at the edges of the weak background.

2.3.6 Self-Spatial Mode Filtering

In a practical experiment, it is very rare, if not impossible, for the spatial mode quality of an intense fs laser pulse to be perfect. Thus, during filamentation experiments, the transverse spatial modes of the laser pulse “compete” with one another to form a self focus and it is the lowest order spatial mode that dominates. That is to say, only the lowest order mode inside the pulse will self-focus into a filament. All other higher order modes remain in the reservoir. This is called self-spatial filtering (Liu and Chin, 2007; Chin et al., 2007; Akturk et al., 2007).

We can understand the above phenomenon of self-spatial filtering as follows. When self-focusing starts, all the modes in a pulse will self-focus initially. However, since the lowest order spatial mode has a smallest diameter as compared to the other higher order modes, it will reach the self-focus first according to Eq. (2.1). At and near the self-focus of the lowest order mode, ionization takes place generating a plasma. All the higher order modes while still self-focusing, will encounter the plasma and be diffracted out into the background reservoir before forming self-foci. The result is that the central filament core comes from the self-focusing of a very clean fundamental mode whose divergence is smaller than the original beam divergence.

This self-spatial filtering would have profound impacts on many applications of the filament. One example is the generation of other pulses at other wavelengths through four-wave-mixing inside the filament (Théberge et al., 2006b). The mode of the newly generated pulse is a perfect lowest order mode because four-wave-mixing
is a nonlinear process interacting with the high intensity core without implicating the low power reservoir practically. Thus, only the fluence distribution in the core is reflected in the newly generated pulse (Théberge 2006a). Another manifestation of the self-spatial filtering shown in Fig. 2.17 shows the evolution of the beam patterns during filamentation. The central symmetrical hot spot persists all along the filament.

2.3.7 Self-Phase Modulation, Self-Steepening and White Light Laser (Supercontinuum)

Because of the shortness of the femtosecond laser pulse, one of the consequences of nonlinear interaction with the medium during filamentation is self-phase modulation. The interaction of the high intensity in the self-focal regions with the neutral gas and with the plasma results in the modulation of the phase of the pulse; i.e., self-phase modulation (SPM) (See for example, Boyd, 2003). The consequence is spectral broadening towards both the red and the blue sides. SPM in an optical medium is caused by the temporal variation of the refractive index.

It is understandable that (also according to theoretical calculation) at the self-focus, where self-focusing is balanced by self-defocusing due to the plasma generation, the wave front becomes plane. To simplify the explanation, we assume that the self-focused wave is a plane wave. In this plane wave approximation, the plane wave front at the self-focus is given by the function.
\[ F(z,t) = \exp\{i[\omega_0 t - k z]\} = \exp\left\{i\left[\omega_0 t - \frac{\omega_0 n}{c} z\right]\right\} \tag{2.3} \]

where \( z \) is the propagation distance, \( \omega_0 \), the central angular frequency of the laser.

\[ n \simeq n_0 + \Delta n(t) \tag{2.4} \]

\[ \Delta n(t) = n_2 I(t) - \frac{4\pi e^2 N_e(t)}{2m_e \omega_0} \tag{2.5} \]

Here, \( n_2 I(t) \) is the Kerr nonlinear refractive index of the neutral gas, \( I(t) \) being the intensity. The last term is the plasma contribution (see footnote 1 of Section 2.3.1) where \( N_e(t) \) is the electron density generated through tunnel ionization of the molecules and \( e \) and \( m_e \) are the charge and mass of an electron, respectively. We note that the electron-ion recombination time is normally of the order of many nanoseconds, much longer than the femtosecond time scale of the pulse. Hence, the generated plasma could be considered as static during the interaction with the pulse. The wave enters the optical medium at \((z = 0, t = 0)\). At the position \( z \), Eq. (2.3) becomes

\[ F(z,t) = \exp \left\{ i \left[ \omega_0 t - \frac{\omega_0 n_0}{c} z - \frac{\omega_0 \Delta n(t)}{c} z \right] \right\} \tag{2.5a} \]

\[ = \exp \left\{ i \left[ \omega_0 t - \frac{\omega_0 n_0}{c} z \right] + \int_0^t \frac{\partial}{\partial t} \left( -\frac{\omega_0 \Delta n(t)}{c} z \right) dt \right\} \tag{2.5b} \]

\[ \equiv \exp \left\{ i \left[ \omega_0 t - \frac{\omega_0 n_0}{c} z \right] + \int_0^t (\Delta \omega) dt \right\} \tag{2.5c} \]

where

\[ \Delta \omega = \frac{\partial}{\partial t} \left( -\frac{\omega_0 \Delta n(t)}{c} z \right) = -\frac{\omega_0}{c} z \frac{\partial [\Delta n(t)]}{\partial t} \tag{2.6} \]

SPM is the modulation (variation) of the phase of the wave due to the self-generated extra phase \( -\frac{\omega_0 \Delta n(t)}{c} z \) (see Eq. 2.5a). It is manifested by the frequency shift \( \Delta \omega \) of Eqs. (2.5c) and (2.6). Since the front part of the pulse always sees a neutral gas, from Eqs. (2.6) and (2.5), without the plasma contribution,

\[ \Delta \omega = -\frac{\omega_0 z}{c} \frac{\partial [\Delta n(t)]}{\partial t} = -\frac{\omega_0 z}{c} n_2 \frac{\partial I(\text{front part})}{\partial t} < 0 \tag{2.7} \]

The last inequality in Eq. (2.7) arises because the front part of the pulse has a positive temporal slope whose value ranges continuously between zero and a maximum value. Hence, the front part of the pulse contributes principally to red (Stokes) shift/broadening. But the back part of the pulse should also see the neutral
gas since the gas is only partially ionized. SPM in the neutral would lead to a blue shift/broadening but this blue shift is masked by the stronger blue shift/broadening due to SPM in the plasma together with the eventual SPM, due to the very steep descent of the back part of the pulse (i.e., self-steepening, Gaeta, 2000; Aközbek et al., 2001).

The contribution of the plasma term in Eq. (2.5) to frequency shift/broadening starts soon after the plasma is generated. The plasma interacts at the self-focal zone with the self-focusing slice of the pulse and with the slices coming from behind (i.e., from the back part of the pulse). Using Eqs. (2.5) and (2.6), the frequency shift/broadening due to the plasma term is

\[ \Delta \omega = \frac{2 \pi \kappa e^2}{c m_e \omega_0} \frac{\partial N_e}{\partial t} \]  

(2.8)

The electrons are generated through tunnel ionization of the air molecules. For simplicity, we can use the experimental results to empirically state that the effective tunnel ionization rate of nitrogen (and oxygen) molecules is proportional to the intensity raised to the power of \( \eta \), where \( \eta \) is the empirical slope of the experimental ion yield versus intensity curve in the log-log scale (Talebpour et al., 1999a). Thus, by solving the rate equation for the generation of free electrons by tunnel ionization, the electron density can be expressed as (Liu et al., 2002)

\[ N_e \approx N_0 w \int_{-\infty}^{t} I^\eta(t) dt \]  

(2.9)

Equation 2.9 is derived from the rate equation of tunnel ionization generating the electron density \( N_e \) in a gas, namely,

\[ \frac{\partial N_e(t)}{\partial t} \propto N_0 I^\eta \]  

(2.9a)

i.e., the rate of change of the electron density is proportional to \( N_0 \), the density of the neutral air and to \( I^\eta \). The latter is the empirical \( \eta \)th power relation of the intensity \( I \) obtained from experiments. Integrating Eq. (2.9a), one obtains Eq. (2.9) where \( w \) is the proportionality constant sometimes called the “cross section” of tunnel/multiphoton ionization. Substituting Eq. (2.9) into Eq. (2.8), we obtain

\[ \Delta \omega = \frac{2 \pi \kappa e^2 N_0 w}{c m_e \omega_0} I^\eta(t) \]  

(2.10)

This positive blue shift/broadening of the frequency is large partially because of the highly nonlinear dependence on the high intensity inside the self-focal region. Besides SPM, towards the end of the propagation; i.e., towards the end of the diffraction length given by \( ka^2 \) (\( k \) is the wave number and \( a \), the beam radius at 1/e value), the back part of the intensity distribution of the pulse becomes very steep and the
slope is negative. This temporal variation happens mostly in the neutral gas because it is at the end of filamentation where ionization is negligible. It would give rise to a large blue shift of the frequency since by analogy to equation 2.7

\[
\Delta \omega = -\frac{\omega_0 z}{c} \frac{\partial I}{\partial t}\ \text{(very steep back part with negative slope)}
\]  

This is a major source of the large broadening towards the blue side of the pump frequency. This so-called self-steepening in the case of the propagation of a powerful femtosecond pulse is the consequence of a continuous spatio-temporal self-transformation process of the pulse during propagation. Numerical simulation (see Section 4) shall give a quantitative picture of what happens (Gaeta, 2000; Aközbek et al., 2001). Here, we shall give a very qualitative, simple minded picture so as to grasp a physical feeling of self-steepening. Group velocity dispersion and the interaction with the plasma are neglected. The central slice of the pulse with the highest intensity where ionization occurs would propagate with a velocity 

\[
c = \frac{n_0 + \Delta n_{Kerr} - \Delta n_p}{n_0}
\]

where \( n_0 \), \( \Delta n_{Kerr} \) and \( \Delta n_p \) are the indices of refraction of the neutral air, the nonlinear Kerr index and the index of the plasma, respectively. At the intensity clamping position, \( \Delta n_p = \Delta n_{Kerr} = n_2 I \). This focusing central slice with intensity clamping would thus have an index of refraction \( n_c = n_0 \) where the subscript \( c \) indicates ‘central slice’: it would propagate faster than the front part of the pulse which sees an increase of the index of refraction due to the nonlinear contribution (Kerr nonlinear index) only \( n_f = n_0 + \Delta n_{Kerr} \) where the subscript \( f \) indicates front ‘part’: i.e., no plasma generation yet. Now the back part of the pulse sees a weak plasma generated by the peak of the pulse. The index of refraction of this plasma zone is the combined values of the neutral and the weak plasma; i.e., \( n_b = n_0 + (\Delta n_{Kerr})_b - \Delta n_p \); here, the subscript \( b \) indicates ‘back part’, \( (\Delta n_{Kerr})_b < \Delta n_p \) because the intensity of this back slice is weaker than the clamped intensity while the plasma is left behind by the clamped intensity. Hence, \( n_b < n_c < n_f \); i.e., the back part of the pulse would propagate faster than the front part of the pulse. Soon, the back part would almost catch up with the front part resulting in a steep rise in intensity at the back part. SPM is proportional to the derivative of this part of the pulse, hence a very large blue shift according to Eq. (2.11).

The propagation distance \( z \) also plays a role in both the red and blue broadening (see Eqs. 2.7 and 2.10). Thus, during experimental observations, the spectral broadening of the pulse develops progressively as the propagation distance increases. Both experiment and numerical simulation (Gaeta, 2000; Aközbek et al., 2001; Kandidov et al., 2003a) show similar broadening. A strong broadening in air towards the red up to 4 \( \mu \)m was recently reported (Kasparian et al., 2000b). The central part of the pattern of Fig. 2.1c, d are examples of such frequency broadening from the pump at 800 nm towards the blue side across the whole visible frequency range; hence, it appears white. This is what we call the self-transformed white light laser pulse.
2.3.8 Conical Emission

The colorful rings in Fig. 2.1a are another manifestation of self-phase modula-
tion in the radial direction. The previous section describes self-phase modulation
in the plane wave approximation; i.e., in the \( z \)-direction only. That is to say, we
have considered only the wave vector \( k_z \). However, the laser pulse front is curved at
the self-focal zone. It contains a transverse part of the wave vector. In a normally
spherically symmetric pulse front, the general wave vector \( \vec{k} \) is given by

\[
\vec{k} = k_z \hat{z} + k_r \hat{r} = k_{0z} \hat{z} + \Delta k_z \hat{z} + k_{0r} \hat{r} + \Delta k_r \hat{r}
\]  
(2.12)

where the initial wave vectors contain a subscript zero. \( \hat{z} \) is a unit vector in the prop-
gagation direction; \( \hat{r} \) is a unit vector transverse to \( \hat{z} \). In the plane wave approximation
described above, we have considered only the \( z \)-components. They are:

\[
k_{0z} \hat{z} = \frac{\omega_0 n_0}{c} \hat{z}
\]
\[
\Delta k_z \hat{z} = -\frac{\omega \Delta n}{c} = \int_0^t \frac{(\Delta \omega)}{z} dt \hat{z} \quad \text{(from Eq. 2.5.1 and 2.5.3)}
\]
\[
= \begin{cases} 
-\frac{\omega_0}{c} n_2 \int_0^t \frac{\partial I}{\partial r} dt \hat{z} < 0 & \text{(in neutral gas)} \\
+ \frac{2\pi e^2}{cm_e \omega_0} \int_0^t \frac{\partial N_e}{\partial z} dt \hat{z} > 0 & \text{(in plasma)} 
\end{cases}
\]  
(2.13)

\[
= \begin{cases} 
-\frac{\omega_0}{c} n_2 \int_0^z \frac{\partial I}{\partial z} dz \hat{z} < 0 & \text{(in neutral gas)} \\
+ \frac{2\pi e^2}{cm_e \omega_0} \int_0^z \frac{\partial N_e}{\partial z} dz \hat{z} > 0 & \text{(in plasma)} 
\end{cases}
\]  
(2.14)

\[
\equiv \begin{align*}
\Delta k_z \hat{z} & \quad \text{(neutral) < 0} \\
+ \Delta k_z \hat{z} & \quad \text{(plasma) > 0}
\end{align*}
\]  
(2.15)

We see that from Eqs. (2.13) to (2.14), the temporal rate of change has been
transformed into a spatial rate of change by recognizing that \( z = ct \). Figure 2.18a
gives a schematic relationship of these waves vectors. From eq. 2.15, \( \Delta k_z \hat{z} \) (plasma),
being positive, is in the same direction as that of the original vector \( k_{0z} \hat{z} \) while \( \Delta k_z \hat{z} \)
(neutral), being negative, is in the opposite direction to that of \( k_{0z} \hat{z} \). Equation (2.14)
(together with Eq. 2.8) shows that the spatial gradient of the electron density gives
rise to a blue shift of the frequency in the \( z \)-direction (plane wave approximation).
Since electrons are generated in the 3-D self-focal volume, electron density gra-
dients show up in all directions; i.e., in both the \( z \)- and the \( r \)-directions. In the \( r \)-direction, the electron density gradient would give rise to a spatial divergence
of the radiation. Thus, the wave vector, \( \Delta k_r \hat{r} \) (plasma), which is in the direction of
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$k_{\text{r}0}\hat{r}$, would make this blue shifted radiation diverging into a ring as shown by the vector diagram in Fig. 2.18b. The larger the electron density gradient is, the larger will be the wave vector $\Delta k_r\hat{r} \sim \frac{\partial N_e}{\partial r} \hat{r}$ and the blue shift (Eqs. 2.8 and 2.14). The radial electron density gradient varies continuously from zero to a maximum value. The detailed analysis of conical emission is given in Kosareva et al. (1997b) and Kandidov et al. (2003a). From Fig. 2.18b, the divergence of the resultant vector $\vec{k}$ will be larger when the electron density gradient is larger. Hence, rainbow-type colored rings are generated around the central white spot; the larger the frequency shift is (i.e., the shorter the shifted wavelength is), the larger the divergence will be. The rings in Fig. 2.1d are thus explained. The frequency shift due to the neutrals would not give rise to rings because the wave vector $\Delta k_r\hat{r}$ (neutral) points in the opposite direction of $k_{\text{r}0}$; i.e., it tends to reduce the divergence of the wave. When the above description was done numerically in 3-D calculation, the agreement between theory and experiments is excellent. Other theories such as four-wave-mixing and Cherenkov radiation have been proposed but they fail to satisfy all the experimental observations (Kandidov et al., 2003a).

Before ending this section, we comment on the repeating dark circular rings in the conical emission (Fig. 2.1d). It is due to the spectral interference (see for example: Chin et al., 1992) of the conical emissions coming from two to three on-axis filaments due to multiple refocusing (Fig. 2.5). Assume that there are two sets of conical emission from two re-focusing filaments along the same axis. Because each color of a conical emission is generated continuously along a filament, the width of each color on the observation screen is wide. Hence there will be overlap of the same color coming from the two filaments. At the overlapping positions, those colors satisfying the condition $nd = (m+1/2)\lambda$ ($d =$ path difference between the conical light of the same color coming from two different filaments, $\lambda =$ wavelength,
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$n = \text{index of refraction, } m = \text{integer}$ would give rise to destructive interference (dark rings), while those satisfying $nd = m\lambda$ would give rise to constructive interference. If there is a bunch of filaments occurring around the propagation axis, there will be so many overlapping rings that the whole conical emission becomes a blur of continuous colored band with no dark rings. This situation can be achieved by passing a sub-terawatt pulse through a piece of BK7 glass without external focusing (Fig. 2.4). In this case, the peak power (300 GW) is much higher than the critical power for self-focusing in glass (a few megawatts) so that multiple refocusing as well as multiple filaments occur around the propagation axis.

2.3.9 Ring Structure at the Pump Wavelength

Conical emission has become an accepted name for the colored rings surrounding the filament axis as described in the previous Section 2.3.8. However, the fundamental wavelength also exhibits ring structure because of the same reason for the creation of the colored conical emission; i.e., spatial gradient of electron density around the filament. But it is not trivial to observe these rings at the pump wavelength because it is masked by the background reservoir at the same wavelength. If one uses an interference-filter to try to filter out the other wavelengths, what is left behind would be the stronger background reservoir with an irregular fluence distribution which might overwhelm the ring pattern at the pump wavelength. The author and his co-workers (Chin et al., 2002) have succeeded in doing such a measurement in air at a distance of more than 90 m away. The background reservoir, after propagating this long distance, has spread out significantly and the rings become evident. This is shown in Fig. 2.19. Note that the central white spot is the over-exposed central part of the pulse at the pump wavelength around 800 nm.

Another manifestation of the ring structure is seen by focusing the beam in air onto a piece of glass and observing the burn pattern. Damage ring structure was observed. This was confirmed by simulation (Chin et al., 2001). Figure 2.20 shows the calculated fluence distribution across the diameter of a filament in air just before the geometrical focus of a lens. Rings are evident. There is a dip at the center. These are due to the diffraction of the plasma inside the filament. Care should be taken in the interpretation of this dip. It is the fluence, not the peak intensity of the pulse. This fluence is expressed as the energy per unit surface of the cross section of the laser pulse and is the temporal integration of the intensity distribution of the pulse. We shall come back to this when we describe the theoretical simulation of the phenomena during filamentation.

2.3.10 Self-Pulse Compression

In recent years, there is a surge of interest in the compression of a laser pulse down to the single cycle level for the sake of generating single attosecond laser pulse
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Fig. 2.19 Ring structure at the pump wavelength taken at a distance of more than 90 m away from the laser output. The central part is over exposed. The structure at the lower left hand corner is another filament that is starting to grow and interfere with the central filament. This will be dealt with in Chapter 4 on multiple filamentation.

through the generation of high order harmonics in gases. It was discovered that the easiest way seems to be the technique based upon filamentation of a laser pulse (Couairon et al., 2005). This concept of self-pulse compression can be understood from the slice-by-slice self-focusing model. It is already apparent in Fig. 2.9. The front part of the pulse becomes narrower and narrower as the pulse propagates. The back part of the pulse spreads out into the broad background reservoir. At any position of the filament or after the filament, if one intercepts the pulse, one would obtain a short pulse (front part in this picture, but could also be the back part which could only be shown in numerical simulation; see later in the theory chapter) whose duration is shorter than the initial pulse duration and could become very short (one cycle). However, there is this background reservoir that has to be taken care of. Otherwise, the pulse would not be a “clean” pulse. More vivid pictures of such pulse compression will be shown in the chapter where theory and numerical simulation are discussed.
2.3.11 \textit{X-wave}

X-waves provide another way of presenting conical emission, supercontinuum, pulse splitting and pulse shortening in the near field ($r,t$) and far field ($k,\lambda$) plots of the intensity contours of the pulse at various positions of propagation during single filamentation. Here $r,t,k,\lambda$ are the parameters pertaining to the pulse: radius, local time, wave vector (or cone angle) and wavelength. The intensity scale is logarithmic which amplifies the contrast in the plots thus showing some X-shaped features. Interested readers are referred to Couairon et al. (2006) for the theory and Xu et al. (2008) for an experimental manifestation of the X-wave and references therein. The physics has already been explained in the “natural” way (i.e., without using X-wave presentation) by Kandidov et al. (2003a). It will be examined in more detail in Section 3.5.

2.4 Full Evolution of a Single Filament

An experiment together with numerical simulation was done in which the full evolution of a single filament in air was measured (Chen et al., 2007). We find that the evolution of the so-called single filamentation starts with the “efficient ionization” zone. The efficient ionization zone is the usual one we measured routinely in the
2.4 Full Evolution of a Single Filament

laboratory (Figs. 2.5 and 2.14). It is followed by the weakly ionized zone that was never observed until recently (Eisenmann et al., 2007). It was also predicted and experimentally (but indirectly) confirmed by Akturk et al. (2007). At the end of this weakly ionized zone comes the quasi-linear propagation of the pulse whose peak power is now lower than the critical power for self-focusing but whose intensity is still high enough to self-focus. However, this self-focusing is overcome by the linear diffraction of the pulse so that the divergence is very small, many times smaller than the normal low divergence of a single mode laser. After that, the pulse propagates linearly.

The experiment was done by using a collimated 10 Hz, 45 fs, 3.2 mJ, 800 nm Ti-sapphire laser beam. This beam self-focused in air at a distance. The beam diameter was measured at different positions along the propagation axis. Since the intensity was high at the self-focusing points, a wedge intercepted the beam at a near grazing angle along the filament. This ensured that the beam area on the surface of the wedge is large and hence the intensity low, thus avoiding damage of the surface. Through partial reflection, the beam pattern at the surface of the wedge was imaged onto a calibrated CCD camera with calibrated attenuation and filtering. The fluence distribution of the pattern can be obtained from the signal of each pixel from each image at each position. The beam diameter at FWHM can then be quantitatively defined. Meanwhile, the nitrogen fluorescence at different positions from inside the filament was measured from the side using appropriate UV filters and a PMT. Figure 2.21 shows the detail of the experimental setup.

![Figure 2.21](image)

**Fig. 2.21** Experimental set up to study the full evolution of single filamentation in air

Figure 2.22a–h shows the evolution of the beam’s transverse pattern along the propagation axis. The central part of the beam stays round throughout the filament zone ($z = 200–700$ cm) and is always surrounded by the reservoir. This is because during filamentation, the initial pulse undergoes self-spatial filtering (Théberge et al., 2006b, 2007a; Chin et al., 2007; Liu and Chin, 2007).
Fig. 2.22 Some typical pictures taken by CCD camera along the propagation axis. $T$ is the transmission of the beam before reaching the CCD camera (see Fig. 2.21). The distance in cm shown below each pattern indicates the distance $z$ from the negative lens (Fig. 2.21). The high attenuation in (c) - (e) renders the background reservoir ‘invisible’.

Figure 2.23a (triangular symbols) shows the fluorescence signal (log scale) as a function of $z$. The fluorescence signal underwent a strong decrease by two orders of magnitude from $z = 250$ to $450$ cm, followed by a decrease by one order of magnitude from $450$ to $690$ cm. From $690$ to $700$ cm, the signal dropped sharply and fluctuated at noise level thereafter.

The evolution of the beam diameter (FWHM of the fluence distribution) is shown in Fig. (2.23b). A sharp decrease of the diameter until about $190$ cm is followed by a slower one. The minimum of $0.18$ mm is reached at around $300$ cm. The beam diverges very slowly from a diameter of $0.25$ to $0.37$ mm at $450 < z < 700$ cm. This is the zone where the fluorescence (ionization) is weak (Fig. 2.23a).

After the filament zone, the laser pulse propagates with a divergence of $0.23$ mrad (FWHM) between $900$ and $1200$ cm (see inset of Fig. 2.23b). This is a very small divergence since linear diffraction from the measured diameter of $370 \mu$m at $z \approx 700$ cm would lead to a much larger divergence of $\lambda/\pi w_0 = 1.6$ mrad assuming Gaussian propagation where $w_0$ is the beam waist at $1/e^2$ of the peak intensity. This is because the on-axis intensity is still high after $z = 700$ cm and the pulse could still self-focus. But since the peak power is smaller than the critical power for self-focusing, there will be no collapse of the pulse. Linear diffraction together with GVD is stronger than self-focusing. The consequence is that the beam diverges slowly. The intensity in this zone soon after filamentation is estimated to be of the order of $10^{10}$ W/cm$^2$ by using the measured data described above.

Numerical simulation (Fig. 2.24) shows that at $200 < z < 300$ cm, the leading sub-pulse (peak power $\approx 3P_{cr}$) dominates (Fig. 2.24b, $z = 252$ cm). For $z > 300$ cm, the leading sub-pulse depletes; the trailing sub-pulse sustains the core energy; the
2.4 Full Evolution of a Single Filament

Fig. 2.23 (a) Measured nitrogen fluorescence signal (open triangle) and energy evolution of the central hot spot at the fundamental wavelength (solid circle) as a function of the propagation distance. Inset: Spectrum at position 270 and 300 cm. (b) Measured beam diameters (FWHM, averaged for horizontal and vertical dimension) as a function of the propagation distance (solid triangle) and the numerical simulations (solid line). Inset: Divergence of a nonlinear propagated beam (with filament, solid circle)

intermediate part diverges strongly due to the plasma left behind by the leading sub-pulse (Fig. 2.24b, $z = 308\text{ cm}, z = 510\text{ cm}$). As a result, the core energy decreases, while the peak power still exceeds the $P_{cr}$. The calculated peak intensity in the region 200–450 cm is clamped to around $5 \times 10^{13} \text{ W/cm}^2$ and the peak electron density is $\sim 0.3 \times 10^{-4}$ of the atmospheric density. The filament is represented by the “efficiently ionized” zone until $z \approx 450\text{ cm}$ (Fig. 2.24a). The shaded region in Fig. (2.24a), $450 < z < 620\text{ cm}$ is characterized by the preservation of the peak power
Fig. 2.24  (a) Linear electron density (filled squares) and power in the most intense slice (solid curve) as a function of the propagation distance. Horizontal line shows $P_{\text{peak}}/P_{\text{cr}} = 1$. Dashed zone – weakly-ionized filament. Note non-monotonic power behavior at pulse splitting position $z \approx 300$ cm.

(b) Curves with filled pattern are on-axis temporal intensity profiles at $z = 252$ cm, $z = 510$ cm, $z = 620$ cm; solid curve in each plot (not filled) indicates initial distribution of a Gaussian pulse. Curve, marked by squares, shows pulse splitting at $z = 308$ cm. Note change in the intensity scale for the plots at each propagation distance (courtesy of Olga Kosareva)

above $P_{\text{cr}}$ but dominated by the trailing sub-pulse (Fig. 2.24b). The leading sub-pulse would no longer produce a plasma. In this case, the integrated electron density falls down an order of magnitude within 20 cm (440–460 cm) (Fig. 2.24a). This decrease in electron density is similar to the decrease in fluorescence in Fig. (2.23). We call the filament in this region “weakly ionized”. It is important to note that the defocusing effect by plasma in this zone, the dispersion and the diffraction are still able to dynamically interplay with the self-focusing effect to sustain a self-guided column (Eisenmann et al., 2007). Later on, by $z \approx 700$ cm, material dispersion and diffraction overcome self-focusing (Fig. 2.24a, $z = 620$ cm) and ionization becomes negligible.

We can now understand that the so-called “nonionizing channels” at long distances might well be the local self-spatially filtered fundamental modes of various hot zones that diverge out slowly after the end of the plasma filament.
Fig. 2.25 Self-focusing and filamentation evolution in air. It starts with self-focusing. Before the collapse of the pulse, there is already some weak ionization in the pulse contraction zone. The pulse collapses into a series of self-foci (filament) with efficient ionization. This is dominated by the leading sub-pulse. The loss of energy results in the dominance of the trailing sub-pulse which continues to contribute to a much weaker ionization. At the end, the peak power of the self-filtered fundamental mode is lower than the critical power for self-focusing and it diverges/diffracts out linearly.

The evolution of the so-called single filamentation can now be defined as follows (see Figs. 2.25 and 2.26): pulse contraction in the quasi-stationary regime (and self-spatial filtering) overcoming linear diffraction and material dispersion $\rightarrow$ efficiently ionized filament dominated by the leading sub-pulse $\rightarrow$ weakly ionized filament dominated by the trailing sub-pulse $\rightarrow$ weak self-focusing (no collapse) overcome by linear diffraction and material dispersion (the author calls this “quasi-linear diffraction”) $\rightarrow$ linear diffraction of the self-filtered fundamental mode. There is always the background reservoir accompanying the propagation that masks the self-spatially filtered fundamental mode at the end of the propagation. That is to say, the fundamental mode and the background reservoir merge into one, and one cannot distinguish them anymore in the nonfilamentation zone.

The merging of the background reservoir and the filament core (fundamental mode) after filamentation is universal. That is to say, for a given spatial fluence distribution of a pulse, if one increases the energy inside the pulse linearly without changing any other property of the pulse, and assuming that the change does not induce multiple filamentation, the only change during propagation would be the
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Efficiently ionized zone (dominated by leading sub-pulse)

Weakly ionized zone (dominated by trailing sub-pulse)

Reservoir

Single fundamental mode due to self-spatial filtering; low divergence

White light laser

Fig. 2.26 A schematic illustration of the evolution of the filamentation of a powerful femtosecond laser pulse in air

Since the pulse would always end its filamentation when the peak power is lower than the critical power, it would always end up with the same spatial distribution (merging of the background reservoir and the fundamental mode); it would thus always diverge in the same way.

2.5 Maturity of a Filament

A filament becomes mature when it has undergone the full evolution of filamentation as shown in Fig. 2.26. That is to say, after self-focusing, the pulse evolves into the pre-filament zone, the filament zone and the post-filament zone. The back part of the pulse becomes very steep (self-steepening). It becomes a white light laser pulse. Such an evolution into a mature filament has been studied in dye solutions demonstrating clearly this evolution scheme (Schroeder and Chin, 2004).

2.6 Filamentation Without Ionization

We can now ask the question, “What really is a filament.” So far, it is implicitly defined as being the self-focusing zone along the propagation axis of the laser pulse
2.7 What Is a Filament?

In view of the continuous dynamic processes that take place during the filamentation of a femtosecond laser pulse, a unique definition of a filament might never be found. However, because of the popularity of this new direction of research, it seems that there is a need to standardize a definition of a filament so as to avoid confusion (Chin et al., 2008b).

Intensity clamping is a unique phenomenon during filamentation. The clamped intensity is very stable. Its root-mean-square fluctuation is more than 10 times less than that of the input pulse before filamentation (Théberge et al., 2006b). The author proposes using intensity clamping as a guideline for the definition of a filament. Because intensity clamping is accompanied by plasma generation, it is convenient to define a filament as being the zone where there is plasma generation. Thus, in the context of the current study, the efficiently and weakly ionized zones together constitute the filament.

Because intensity clamping is a unique phenomenon, the author proposes the following definition. A filament is the propagation zone where there is intensity clamping. Since intensity clamping is the consequence of the generation of plasma that balances the self-focusing act of the pulse, this definition would mean that a filament is the propagation zone where plasma is generated. The zones before and
after the plasma zone where the intensity is still relatively high but not sufficient to generate a plasma could be conveniently called pre- and post-filament zones.

Before ending this section, it should be emphasized that self-focusing alone is not always sufficient to produce a filament. To obtain self-focusing, it is sufficient to have an inhomogeneity of intensity across the wave front. But if the peak power is lower than the critical power for self-focusing, or if the divergence of the beam is stronger than the focusing power of self-focusing (Liu et al., 2006a), linear diffraction, GVD and divergence would overcome the self-focusing and there will be no collapse that leads to filamentation.