Resolution Enhancement of Imaging Systems by Quantum Phase Amplification

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Outline

• Quantum Phase Amplification (QPA)

• Numerical Model for Transverse Phase Amplification
  - Transverse Phase Amplification
  - SNR for Two Point Source Distinguishability

• Preliminary Experimental Results and Experimental Design

• Conclusions and Future work
The Classical Imaging Problem

Diffraction-limited
- Rayleigh criterion: $\theta_R = 1.22 \frac{\lambda}{D}$
- Dawes’s limit: $\theta_D = 1.22 \frac{\lambda}{D}$

Classical resolution enhancement:
- Limited control of wavelength
- Synthetic aperture imaging

Quantum Phase Amplification (QPA)

\[ E' = A \left( \cos \theta + i \sqrt{g} \sin \theta \right) \approx \frac{A}{\sqrt{g}} \exp(ig\theta) \]

\[ E = A(\cos \theta + i \sin \theta) = X + iY \]

- Nonclassical state transformation with amplitude squeezed and phase anti-squeezed
- A factor of \( g^2 \) improvement in SNR is possible


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The rate of change of phase results in angular amplification.

Resolution goes beyond Rayleigh limit: \( \Delta \theta = \frac{\lambda}{(g \ D)} \)

Received state quantum control by OPA to enhance classical detector.

\[ \theta_{\text{out}} = g \theta_{\text{in}} \]
Phase Amplification Characteristics

- Degenerate with phase matching
- Ideal phase-amplification condition:
  \[ \phi_{10} + \phi_{20} = \phi_{30} - \pi/2 \]
- Phase amplification characteristic:
  \[ \Delta \phi_{\text{out}} = f(\Delta \phi_{\text{in}}) \]
- A nearly ideal phase amplifier for small departure

Ideal plane wave: no transverse phase variance at the input of the amplifier

\[ \frac{\partial \psi(x,0)}{\partial x} = 0. \]

Plane wave is impossible for transverse phase amplification (TPA)

Transverse Phase Amplification: Multimode Operation

- Two necessary conditions:
  - Spatially varying phase: $\partial \psi(x, 0) / \partial x = 0$
  - Symmetric spatial frequencies

Performance of Multimode TPA

- The linear relationship holds for $G \theta_0/\theta_R \ll 1$
- Closely approximate an ideal PSA for small amplified angles
- Sub-Rayleigh to sub-Rayleigh enhancement, with the max $\sim \theta_R$

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SNR for Detection of Optical Signals

- Assumption one: two point sources with uniform background
- Assumption two: one point source with uniform background
- SNR is to quantify the difference between the two assumptions

\[
\text{SNR} = \frac{(\bar{g}_1 - \bar{g}_0)^2}{\sigma_0^2} = \frac{\left[ \int M(x) \ln \left[ \frac{M_1(x)}{M_0(x)} \right] dx \right]^2}{\int M_0(x) \ln^2 \left[ \frac{M_1(x)}{M_0(x)} \right] dx} = \frac{3(45\pi^2 - 272)}{2(3\pi^2 - 16)} \left( \frac{Hka|u_1|}{2R} \right)^4 D_{\theta_0}^2 H^2
\]

\[
= C \left( \frac{H\theta}{r_0} \right)^4 D_{\theta_0}^2 \frac{H^2}{r_0} = H^2 D_{r_0}^2.
\]

With phase gain H, the SNR is improved by $H^2$

SNR for Scenario I

- Angular separation is achieved
- SNR is zero before PSA
- SNR increases with gain by PSA

**SNR for Scenario II**

- **SNR is zero before PSA**
- **PSA modifies two hypothesis differently**
- **SNR increases with gain by PSA**
Detector Segmentation

Near field

- To optimize SNR by choosing the right ratio of beam size to pixel size
- SNR saturates for fine pixel size
- Oscillation occurs when pixel size is close to beam size

Far field

Hypothesis 1
Hypothesis 2
Preliminary Experimental Results (Temporal)

Small phase amplification was observed

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Experimental Design for Beam Deflection

- Obtain beam angle deflection by measuring interference pattern changes
- Obtain spatial gain distribution

Diagram: Experimental setup with delay lines, BBO crystal, CCDs, and interference patterns.
Conclusions

- Transverse quantum phase amplification goes beyond diffraction-limited imaging

- SNR improves for point source distinguishing problems by applying phase amplifier.

Future work

- Experimental verification of beam deflection

- Experimental design and verification of sub-Rayleigh imaging